CONTENT COMPANION

8TH Grade, Module 3

KIPP EUREKA MATH

1. BIG IDEAS

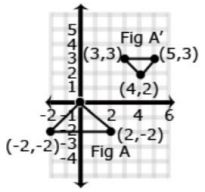
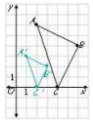
- Dilations magnify and shrink figures.
- Similarity is more than "same shape, same size"; it's a dilation followed by a congruence transformation, or vice versa.
- Rotations, translations, and reflections are abundant in the plane
  - For every point in the plane, there are an infinite number of rotations up to 360°
  - For every line in the plane there is a reflection
  - For every directed line segment there is a translation.
- Scale factor and proportionality can be applied to similar figures to determine missing lengths of similar figures.
- Similarity is transitive.
- Knowledge of similarity and angle-angle criterion allow us to analyze a proof of the Pythagorean Theorem.

2. SUGGESTED LESSONS FOR STUDY

Lesson	*Pay particular attention to:*
3	Navigating the discussion with the dilation factors (magnifying and shrinking via reciprocals)
4	Building the bridge between dilation and similarity. Developing an understanding of the relationship between angles, parallel lines, and transversals. Facilitating a Socratic seminar in math as opposed to a problem set.
9	Stamping the properties of similarity: angle-preserving (from lesson 8), transitive, symmetric, and apply to triangles.
11	Making the conceptual connection from previous lessons, releasing control to students [act as a facilitator]

**Note, Topic C, the proof of the Pythagorean Theorem, or the application of our understanding of similarity, is taught after the End of Module Assessment. If Topic C is not taught within module 3, it is important to ensure that it is taught before Module 7.*

Common Misconceptions	**Ways to Address**	
TOPICS A - C(cont.)		
Misconception	*Conceptual Root*	*Potential Next Steps*
Similarity is the same shape, different size.	• Despite this being a key point of Lesson 1, students have potentially internalized congruency as 'same shape, same size' and similarity as 'same shape, different size.'	• Be mindful of your language. Instead of using shortcuts, define similarity as a dilation followed by a congruence transformation. • Use patty paper to trace pre-images and post-images to highlight difficult to spot distortion. • Ensure that in topic B, you define similarity both in terms of transformations and in terms of measurements of corresponding sides and angles.
Students may look at the scale factor as the distance that is added to the original distance.	• Especially when the dilation is occurring from a vertex, students draw additional segments to the figure to demonstrate the dilation. They often will draw too many (if the scale factor is 2, they will draw 2 additional segments), which would actually be a factor of 3).	• Be sure to have students practice with a scale factor between 0 and 1, so they practice reduction in addition to enlargement. Reduction should challenge their reasoning. • Have students refer back to the center of dilation (in 8th grade, the origin) for both the pre- and post- image.

		• Ensure that students understand scale factor as a ratio (from 6th grade) and a constant of proportionality (from 7th grade).
Common Misconceptions		**Ways to Address**
TOPICS A - C(cont.)		
Misconception	*Conceptual Root*	*Potential Next Steps*
Students struggle to identify scale factor when the sides of the pre-image and image overlap.	They are prone to focus not think of overlapping sides as corresponding sides.	• Direct them to specific side lengths to compare (instead of the ones where there are overlaps, the third side). • Ask students to look for properties relating the sides of the image and pre-image of the dilation. • Provide a variety of dilations for intense student practice, using a range of scale factors (including those less than 1).
Students are unable to visualize a sequence of transformations when orientation is reversed.	Students need to have their threshold of productive struggle developed. They typically look for one step (or one transformation) that will be sufficient.	• Encourage students. • Ask them questions "how might we have moved from the pre-image to the post image" • Describe the series of transformations that results in the transformation of Figure A to Figure A'. 
Students assume a line is straight, or that two lines are parallel, just by looking at them.	This is less a misconception as it is a demonstration of a lack of investment. Students with a low level of investment or productive struggle, will often attempt short cuts (just by looking) without providing mathematical proof. In the past, when two lines have looked parallel, they likely were.	• Ensure that students ALWAYS provide mathematical evidence for their geometric claims. • Always ask students to prove it.
Students apply the scale factor partially (ie, only to the y-coordinates)	This could be rooted in math practice 6 – attend to precision, where students could be working carelessly. This also could be a lack of understanding what is happening to the dilation on the coordinate plane. (For example, in the included diagram, they might only apply the scale factor to the x-coordinate, since C and C' are on the same y-coordinate. 	• Ask: what did you anticipate would happen when you applied your scale factor? Did this happen? • Ask the student to describe the scale factor of the image to the original image/pre-image for each pair of corresponding sides. • Have students manipulate dilations via Desmos or Geogebra, and reflect on what has happened.

Common Misconceptions		Ways to Address
TOPICS A - C(cont.)		
Misconception	*Conceptual Root*	*Potential Next Steps*
Students are unable to explain how transformations can be used to show that the angle-angles similarity criterion works for triangles.	The logic underlying this concept is challenging. It is less likely a misconception so much as a complex concept that requires a lot of practice, and explicit time making connections.	• Provide students with many opportunities to explore what happens when they try to draw 2 different triangles with the same angle measures • Challenge students to observe what happens to triangle angles when they carry out similarity transformations. • Have students determine what measurements are needed to ensure two triangles are similar. • Provide writing prompts to students to explain the connection across these concepts. • Allow students to carry out transformations via Desmos or Geogebra, to 'bring the transformations' more to life.
Students multiply fractional scale factors incorrectly.	Many of our students were taught procedural operations with fractions, and struggle to apply those procedures accurately, especially in a new context.	• Have students determine if their dilation should be enlarging or reducing, and use that to assess the reasonableness of their answer.

4. PITFALLS TO AVOID

- Teaching topics A and B in isolation
- Defining the center of dilation always at (0,0)
- Not giving students enough practice exploring how and why these properties work (focusing on teaching the proofs).
    - Students need to construct the properties of dilation, similarity, and the Pythagorean Theorem, not be told them.
    - Do not front-load the properties or proofs
    - *"Writing a formal proof of a geometric result is the endpoint of a significant piece of mathematical investigation. It is not generally an activity to be undertaken on its own. In particular, it occurs after an invariance has been detected, conjectured, and tested against a context of variation; after an appropriate diagram has been constructed and understood; and after relevant definitions have been brought into play." NCTM (2013)*

6. ADDITIONAL INFORMATION

Geometry Progressions Document provides heavy content and vertical alignment support. Written by the writers of the Common Core Standards, it provides helpful guidance around the role this content plays in 8th grade, especially in terms of preparing students to be successful in high school content.

Identifying Similar Triangles is a Formative Assessment Lesson which targets Similarity. Unlike Eureka, this highlights Angle sum, exterior angle, and the use of transversals. It would be beneficial to use as either extra practice, or to see students apply knowledge in a different context to demonstrate mastery.

Explore Desmos and Geogebra as websites which can simulate these actions.

In 8th grade, students only work with dilations from the origin. As a result, students will often attempt short cuts (just by looking) without providing mathematical proof. Many dilations that 8th graders are exposed to also undergo rigid motions, so the center of the dilation is not always evident at first glance. Be sure students are always providing mathematical evidence for their claims.

5. HELPFUL HINTS

- Practice not only the math problems in each lesson, but the vocabulary behind the lessons → what does "first consequence of FTS" mean? How can you state the converse of the FTS in your own words? Ensure you can efficiently and naturally navigate the language.
- Make sure that you can explain why Eureka's dilation instruction starts with points, then moves to rays, lines/segments, angles, and finally figures.
- Have access to a lot of graph paper and patty paper. You can also see if there are old stores of transparency paper and/or can use parchment paper if you don't have access to patty paper.
- Be consistent and relentless about ensuring mathematical evidence for any geometric claim.
- Make time for topic C in this unit. While Eureka allows for it to be taught preceding module 7, students will have a better understanding of the roots of the Pythagorean Theorem if its first explored in conjunction with dilation and similarity.