CONTENT COMPANION

5TH Grade, Module 1

KIPP: EUREKA MATH

1. BIG IDEAS

- Ten is still special.
- Place value understanding is the foundation for rounding, comparing, and performing all 4 basic operations with decimals.
- The patterns and relationships that exist between whole number place values hold true for decimals.
- We can use multiplication to solve problems when we have multiple, equal-sized groups; we can use division to find the size or number of equal-sized groups.

2. SUGGESTED LESSONS FOR STUDY

Lesson	*Pay particular attention to:*
3	Language around explanations of patterns and the meaning of exponents
7	Number line model; how the teacher thinks aloud about the thinking work (this is a great thing to practice!)
13	How we can use units, bundling and unbundling to divide; area model for division

3. Common Misconceptions		Ways to Address
TOPICS A - C		
Misconception	*Conceptual Root*	*Potential Next Steps*
As students discover the patterns that result in multiplying by 10 or $\frac{1}{10}$, they often characterize it as "adding a 0" and "taking away/crossing out a zero."	Students have likely learned the "trick" of counting zeroes, and not the conceptual understanding that when we multiply by tens, the digits shift to the left, leaving place values "open." We place a 0 there to show there are no more of that place value.	• When a student says this, push them to explain why, using place value reasoning. • Be careful not to casually use this language yourself.
With decimals, students often think that longer numbers mean larger numbers.	This is the case with larger numbers, and therefore, students may be using the length of the number to compare whole numbers rather than the understanding that longer numbers are larger because they have more of larger place values.	• Reinforce that when we add more of larger units, the number gets larger and more of smaller units, the number gets smaller. • Give them opportunities to explain, using place value reasoning, why this not true (problems like, "Jasmine said that 24.509 is larger than 24.59 because it's longer. Explain why Jasmine is incorrect.") • Ask, "If longer whole numbers are larger, what happens if the numbers are the same "length"/have the same number of digits? How do we tell then?" This will get them to dig into the concept, which they then should apply to ALL numbers.

Common Misconceptions		Ways to Address
TOPICS A - C(cont.)		
Misconception	*Conceptual Root*	*Potential Next Steps*
Thinking that if 1 hundred if larger than 1 ten, then 1 hundredth must be larger than 1 tenth.	Students are relying on whole number place values rather than on their understanding of unit fractions and compositions of unit fractions to inform their understanding of fractional place values.	Briefly review 4th grade work around the meaning of the unit fractions $\frac{1}{10}$, $\frac{1}{100}$ and $\frac{1}{1000}$ to access prior knowledge before getting into Lesson 1 of 5th grade.
Thinking that "the decimal moves" when we multiply by a power of 10; the decimal doesn't move – it's always between the ones place and the tenths place. The digits move places depending on their value.	There's not a conceptual root here so much as carelessness with language and relying on a "trick" for ease. It's easier to move a decimal than to move digits in writing.	Reinforce this with your own language – the digits move, not the decimal – and hone in on/correct it when your students say, "The decimal moves…" Note: It's fine to move the decimal as an efficient way of showing the moving of digits, we just don't want to talk about it that way. It's congruous with moving the place value chart instead of the digits, which we could do, as long as we understand it's a way of showing the shifting digits.
Students line up addition and subtraction problems on the left or right rather than lining up place values.	Students are not thinking of the digits in the number as individual units, or don't understand that when they add and subtract numbers, they are adding and subtracting units.	• As you are teaching this, have students continually justify why we are lining our numbers this way. • Have them try it the "wrong" way, or show work done the "wrong way" and explain why it doesn't' make sense. **Note:** We really want to hone students' habit of thinking of the composition of units in a number as they think of the number itself. (i.e., When I see 504, I ALSO see 5 hundreds 4 ones)
Students may struggle with finding the halfway point between decimal place values for rounding.	Students are likely "clinging" to steps like underlining a place and circling the number to the left, and a rule like, "More than 4, add one more." When students are focused on this, they are not practicing finding the halfway point, including using place value reasoning to find it.	Have students think of the numbers in unit form and relate to whole numbers (halfway between 30 and 40 is halfway between 3 tens and 4 tens, which is 3 tens 5 ones; halfway between 0.3 and 0.4 is halfway between 3 tenths and 4 tenths, which is 3 tenths, 5 hundredths)
Students often don't understand why we use the standard algorithm for adding and subtracting (bundle/unbundle) from left to right, when we compare from right to left.	There's not really a conceptual misunderstanding here; in fact, we want students to understand that adding and subtracting is just combining and taking away to find totals, and I can combine in any order I like. If I have 4 gummy bears and 8 twizzlers and my friend has 3 gummy bears and 12 twizzlers, I can combine the gummy bears first to find the total or the twizzlers first to find the total. It's the same with place value. What	Ask questions during instruction like: • When we look at a subtraction problem, what tells us we are going to need to unbundle? • Can we add place values from left to right (the answer is yes!)? What does our number often look like when we do this? (double digits in our place values) How can we take care of that while we

	likely hasn't clicked for students is that the algorithm is efficient because it gives us a way of bundling WHILE we are doing the problem. From left to right, you can only combine totals and then go back and change to standard form; from right to left, we can bundle while we are doing the problem, making it much more efficient.	are adding/subtracting instead of at the end? (add from right to left; then we can bundle as we go instead of at the end) • Carmen has \$15.67 and Jose has \$17.65. Carmen says she has more money because she has 67 cents and Jose only has 65 cents. Jose says that's not true because he has \$17 and Carmen only has \$15. Who's right? Why?
• With expanded form, students sometimes think that the power of 10 indicates the location of the digit (6×10^3 as 600 because 6 is in the "third spot", for instance) rather than understanding it as an indication of the number of 10s that were multiplied in 10^3. • On the flipside, we don't want students only thinking of the exponent as how may zeroes the number has. (10^3 is a 1 with 3 zeroes); grounding the exponent in the meaning helps students distinguish between the meaning of an exponent and a coefficient later on.	This typically happens when: • students have gotten in the habit of thinking/saying "adding zeroes" rather than multiplying by tens. • Students aren't making meaning of expanded form. Students can often think of putting a number in expanded form as "steps" (I put the digits here, $\times$ and $+$ here, and the place value here) rather than showing the meaning of a number (this number has 6 thousands and 4 hundreds; I can write this as $6\times1000+4\times100$).	• Provide place value charts that have (or where students write) each place value written as a power of 10. • As you show student work with numbers written in multiple, equivalent ways, be sure to include exponent notation. • When they are first learning this, or for those that are struggling, have students write out a number given in expanded form with exponents and in expanded form with numerals (for instance, $6\times10^3+4\times10^2+5\times10+8\times1$ as $6\times1000+4\times100+5\times10+8\times1$) • Focus on the **meaning** of this form (this helps with the decimal place values, too)

Common Misconceptions		**Ways to Address**
TOPICS D - F		
Misconception	*Conceptual Root*	*Potential Next Steps*
The division algorithm can be tough for a lot of students, especially if they struggled with it with whole numbers in 4th grade.	This is a particular struggle when students don't think of division as both the inverse of multiplication and as repeated subtraction, and when they don't think of place value as units and multiplication as "groups of."	Use the Eureka way to think about and model it; this approach is rooted in place value, in "groups of" language, and in the relationship to multiplication. (see Grade 4, Module 3, Topic E)
Students want to line up decimals (place values ☺) when multiplying (the truth is that it doesn't matter if we line up place values or not in multiplication). What we want students to understand is the thinking: we multiply each place value in one number by each place value in the other.	• Students are not understanding multiplication as a groups of x. • Students are not thinking of multiplication in parts: multiplying 3 by 4.8 means 3 multiplied by 4 (3 **groups of** 4) and (combined wih) 3 multiplied by 0.8 (3 **groups of** 0.8) – better yet, 3 groups of 8 tenths, which is 24 tenths!	Revisit the area model with two whole numbers, a decimal by a whole number, to activate prior knowledge. Use "groups of" and place value language and talk about **what is happening** when we multiply. Then, when you get to the algorithm, they understand the process and you can just tell them, "We set it up this way." It's not the end of the world if they don't. Algorithms come from and represent the (efficient) thinking and we don't have to notate in a certain, specific way.

4. PITFALLS TO AVOID

- Lack of persistence with place value and units language even as we go into notation with carrying and borrowing. (need to keep saying "2 tens minus 3 tens" and "I don't have enough tens")
- Reducing bundling ten of a unit or unbundling a unit to "carrying over" or "borrowing from a neighbor." (need to say "I need to unbundle 1 of my hundreds into 10 tens" and "now I have 3 tens")
- Talking about lining up decimals rather than lining up place value.
- Presenting rounding as a skill-based procedure rather than a tool for estimating for reasonableness.
- Not making explicit connections of tenths to unit fractions, which students have been learning since 3rd grade.
- Using "shortcut" language: 10^3 is a "1 with 3 zeroes", when we multiply by 100, we "add 2 zeroes", the decimal moves to the left/right, etc.
- Not framing rounding in the context of estimation and giving it a purpose.

6. ADDITIONAL INFORMATION

- **Rounding:** We want students to be able to estimate for reasonableness and also to decide upon necessary levels of accuracy for specific situations. We teach them to round by place value to explore levels of accuracy (smaller place values = closer to actual answer), but long-term, students will not have to round to specific place values; they will need to use estimation to assess reasonableness, or to give information with less, but appropriate, specificity (there were about 80,000 people at the football game; I need about 200 bottles of water for everyone to have at least 1 bottle and enough if a few extra people show up, etc.) in a meaningful way.
- **Standard Algorithm:**
  - In the standards, standard algorithms are the culmination of work with a specific operation. Students use lots of strategies based on place value to gain the conceptual understanding that optimizes efficiency with the algorithm.
  - Standard algorithms for base-ten computations with the four operations rely on decomposing numbers written in base-ten notation into base-ten units. The properties of operations then allow any multi-digit computation to be reduced to a collection of single-digit computations. These single-digit computations sometimes require composition/decomposition of a base-ten unit.

5. HELPFUL HINTS

- Don't skip the fluency activities!
- Spiral in place value concepts around whole numbers and 4th/5th NBT standards in general.
- Continually make connections between multiplication, unit form and standard form $(2 \times 100 = 2\ hundreds = 200)$
- Use place value language even as you are using the algorithms for all 4 operations, and as you are notating with "carrying" and "borrowing"
- When adding decimals, before getting to the algorithm, don't have problems lined up vertically and let students add place values and get more than 10, THEN bundle to make the number in standard form. This will make the connection to the standard algorithm stronger and strengthens mental math skills.

$$24.5\ +\ 3.9\ is\ 2\ tens\ 7\ ones\ 14\ tenths,$$
$$which\ is\ 2\ tens, 8\ ones, 4\ tenths, which\ is\ 28.4$$

- Use the halfway point understanding for rounding (vertical number line is useful); this strengthens students' understanding of relationships between numbers and encourages them to think in unit form. It also facilitates Mathematical Practice 8, *Looking for and expression regularity in repeated reasoning.*
  - Halfway between 1 ten and 2 tens is 1 ten 5 ones
  - Halfway between 1 hundred and 2 hundreds is 1 hundred 5 tens
  - Halfway between 1 tenth and 2 tenths is 1 tenth 5 hundredths
- Use "groups of" language with multiplying. This helps with why we don't need to line up place values. When students understand what they are doing with the numbers under different operations, they understand that we are still doing that when we write them vertically.

$$24.5 + 3 \rightarrow 2\ tens + 0\ tens, 4\ ones + 3\ ones, 5\ tenths + 0\ tenths$$

$$24.5 \times 3\ \rightarrow 3\ groups\ of\ 2\ tens, 3\ groups\ of\ 4\ ones, 3\ groups\ of\ 5\ tenths$$