

**Conceptual Understanding Planning for Ratio Exemplar.** This document serves as a model for planning for ratio. A key component of conceptual understanding is putting the heavy cognitive lifting on students through strategic questioning and the use of ratio techniques.

Example Problem	
<p>During one rainy day on vacation, a family of five decided to go watch a matinee movie in the morning and a drive-in movie in the evening. The price for a matinee movie is different than the cost of a drive-in movie in the evening. The tickets for the matinee movie cost \$6 each and the family spent a total of \$75 going to the movies that day. What was the cost of a ticket for the drive-in movie in the evening?</p>	
1. What background knowledge do students have that will help them access this problem?	2. What is the key information in this problem?
<ul style="list-style-type: none"> <li>- Students are able to translate and solve one step equations</li> <li>- Students are able to apply the distributive property to expressions and equations</li> <li>- Students have worked with tape diagrams in the past (and algebraic tape diagrams)</li> <li>- Students should understand the context of the problem</li> </ul>	<ul style="list-style-type: none"> <li>• There are five people in the family and they are going to two different movies</li> <li>• The cost of the matinee and drive-in are not the same</li> <li>• The cost of the matinee is \$6</li> <li>• We don't know the cost of the evening movie</li> <li>• The total cost for the family for both movies is \$75</li> <li>• We need to find the cost of the drive-in movie</li> </ul>
3. What opening question will you ask to assess students' understanding of the context of the problem? What is the desired answer?	4. How will you ask the opening question? Script your question and ratio techniques.
<p>Q: Tell me about the different movies this family is going to?</p> <p>A: They are going to two different types of movies. A matinee movie and a drive-in movie at night.</p>	<ul style="list-style-type: none"> <li>• "Take 30 seconds and re-read the problem to yourself. At the end of the 30 seconds I want you to be able to tell your partner about the different types of movies this family is going to." (PTT)</li> <li>• "Turn and tell your partner about the different types of movies." (TPS) <ul style="list-style-type: none"> <li>○ As the students talk I'll listen for a pair that has the correct answer. (AM)</li> </ul> </li> <li>• "Let's hear your thoughts..." <ul style="list-style-type: none"> <li>○ I'll randomly select a few groups to share their answers. (PF)</li> <li>○ If I don't end up with the answer I'm looking for I'll call on a group that I know has the correct answer. (PS)</li> </ul> </li> </ul>
5. What is the heavy lifting that you want students	6. What question will you ask that gets students to

<p>to do? What question will you ask? What answer are you looking for?</p>	<p>focus on the information? Script your question and ratio techniques.</p>
<p>HL: Determining the cost of each movie.</p> <p>Q: We know that there are two different types of movies, what do we know about the cost of the movies?</p> <p>A: We know that the cost of the matinee was \$6, and that the total cost of both movies was \$75. We don't know the cost of the drive-in movie.</p>	<ul style="list-style-type: none"> <li>• “Take 15 seconds of think time and go back to the problem if you need to—we know that there are two different types of movies, what do we know about the cost of the movies?” (PTT) <ul style="list-style-type: none"> <li>○ I'll push students to be specific about each piece of information and list the info as they give it. (MC / CS)</li> </ul> </li> </ul>
<p>7. What is the next piece of heavy lifting that you want students to do? What question will you ask? What answer are you looking for?</p>	<p>8. What question will you ask that gets students to focus on the information? Script your question and ratio techniques.</p>
<p>HL: Determining the cost for one family member to go to both movies.</p> <p>Q: What is the cost for one family member to go to both movies?</p> <p>A: It costs one family member \$6 plus the cost of the drive-in (<math>e + 6</math>).</p>	<ul style="list-style-type: none"> <li>• “So you've told me about the different types of movies and the cost of each. What can we say about the cost of one family member going to both movies? Take 20 seconds to see what you can come up with, and then I'll have you compare your results with your partner's. (PTT)</li> <li>• “Compare your answer to your partner's.” (TPS) <ul style="list-style-type: none"> <li>○ I'll circulate and survey the different answers. I'm looking for (<math>e + 6</math>). (AM)</li> </ul> </li> <li>• “Let's hear some different thoughts.” (PF/P) <ul style="list-style-type: none"> <li>○ As students share their different thoughts I'll list them.</li> </ul> </li> </ul>
<p>9. What is the next piece of information you want students to access? What answer are you looking for?</p>	<p>10. What question will you ask that gets students to focus on the information? Script your question and ratio techniques.</p>
<p>HL: I want students to compare the different representations of the cost for one family member and determine the best one.</p> <p>Q: Which representation is the best one for us to use going forward? Why?</p> <p>A: (<math>e + 6</math>) is the best representation because it is a correct algebraic expression.</p>	<ul style="list-style-type: none"> <li>• “Ok—we have a good list of options here. Which is the best option for us to use going forward? Why? Turn and talk to your partner, then be ready to defend your answer.” (TPS) <ul style="list-style-type: none"> <li>○ I'll monitor the conversations and listen for (<math>e + 6</math>)</li> </ul> </li> <li>• “Let's hear some of your thoughts. As your classmates share out, listen carefully to what they are saying. Remember, we're looking for the best representation for us to use. You can add on, agree, or disagree with statements, but be careful not to simply repeat what's already been said.”</li> </ul>

	<p>(HOD)</p> <ul style="list-style-type: none"> <li>○ I'll use minimal words to keep the conversation going and to drive the consensus to <math>(e + 6)</math>.</li> </ul>
<p>11. Continue scripting heavy lifting, questions, and ratio techniques until the problem is complete.</p>	
<p>HL: Student will create a tape diagram that can be used to create an equation and solve the problem.</p> <p>Q: Take the information that we've discussed and create a tape diagram that represents this problem.</p> <p>A: (a correctly drawn tape diagram)</p> <p>-----</p> <p>HL: Students evaluate a displayed tape diagram to see if it is correct.</p> <p>Q: Does this tape diagram correctly display the information from this problem?</p> <p>A: (Yes... with correct reasoning... )</p> <p>-----</p> <p>HL: Students create and solve the equation based on the information in the tape diagram.</p> <p>Q: What equation can we make from the diagram? Why?</p> <p>A: <math>5(e + 6) = 75</math></p>	<ul style="list-style-type: none"> <li>● "We've talked about a lot of information here. Before we start to solve the problem, let's take this information and represent it in a tape diagram. Take 3 minutes to work on this individually." (PTT) <ul style="list-style-type: none"> <li>○ As students work, I'll circulate to look for correct tape diagrams and also work with any students who are stuck. (AM/PS)</li> </ul> </li> <li>● "Let's take a look at ____'s work. Does the tape diagram correctly display the information from the problem?" (SC) <ul style="list-style-type: none"> <li>○ As students answer, I'll push on the specifics of the tape diagram to make sure they can match it up with the context of the problem.</li> </ul> </li> <li>● "We know that we can use tape diagrams to help us create equations. Use this diagram to help you create an equation that can help you determine the cost of the drive-in movie." (PTT) <ul style="list-style-type: none"> <li>○ I'll circulate and observe student work looking for the correct equation.</li> <li>○ Then, I'll poll the class and use HOD to help us determine the best equation to use. (HOD)</li> <li>○ Students will solve the problem individually and we'll put the answer in the context of the problem. (PTT)</li> </ul> </li> </ul>

**Ratio Technique Acronyms Defined:**

Technique	Definition	Effect
Prompted Think Time (PTT)	Giving students time to individually process material or questions before asking for answers or explanations.	<ul style="list-style-type: none"> <li>• Allows for processing time</li> <li>• Engages all students</li> <li>• Students give more well thought-out and articulate answers</li> </ul>
Think/Pair/Share (TPS)	Giving students individual think time, time to discuss with a partner, and then a chance to share with the whole group.	<ul style="list-style-type: none"> <li>• Allows for individual processing time</li> <li>• Partner share offers a chance to articulate an answer out-loud before sharing whole group</li> <li>• Gives the teacher a chance to hear student thoughts before whole class discussion</li> <li>• Gives the teacher a chance to work with individual students</li> </ul>
Aggressive Monitoring / Planting Seeds (AM/PS)	Listening to students' conversations to get a preview of their thinking and/or working with small groups of students to help them shape and articulate their thinking.	<ul style="list-style-type: none"> <li>• Gives the teacher a temperature check of student understanding</li> <li>• Allows the teacher to safely cold call students</li> <li>• Gives the teacher a chance to work with lower level students and help them participate</li> </ul>
Poker Face (PF)	Seeking out other thoughts, ideas, or methods after the first answer is given.	<ul style="list-style-type: none"> <li>• Honors multiple ideas/approaches</li> <li>• Gives more students an opportunity to share their thinking</li> </ul>
Show Call (SC)	Using displayed student work to drive the conversation and questioning.	<ul style="list-style-type: none"> <li>• Students become more intentional and organized with their work</li> <li>• Implies that student work is more important than the teachers' work</li> </ul>
Prompted Habits of Discussion (HOD)	Naming the habits of discussion that students are expected to use before beginning the conversation.	<ul style="list-style-type: none"> <li>• Lets students know exactly what is expected of them in the conversation</li> </ul>
Generalization (G)	Testing the limits of new concepts by changing the situation slightly and/or asking students to apply a concept in a novel setting.	<ul style="list-style-type: none"> <li>• Builds the habit of generalization and testing theories</li> <li>• Builds perseverance</li> </ul>
Strategic Positioning (SP)	Positioning oneself off to the side of the room or among the students.	<ul style="list-style-type: none"> <li>• Takes the focus off of the teacher as the 'leader' of the group</li> </ul>
Inclusive Language (IL)	Using inclusive language such as "we" and "our" when referring to the work and tasks that the class is completing.	<ul style="list-style-type: none"> <li>• Makes it clear that students are important in the problem solving process</li> </ul>
Complete Sentences (CS)	Ensuring that students are using complete and mathematically correct sentences.	<ul style="list-style-type: none"> <li>• Helps students practice mathematical explanations</li> </ul>
Purposeful Pairings (PP)	Strategically pairing students who can work well together and/or help teach each other.	<ul style="list-style-type: none"> <li>• Leads to more productive turn and talks</li> <li>• Can be an added support for lower and high level students</li> </ul>